Zee p. 212
2nd edition

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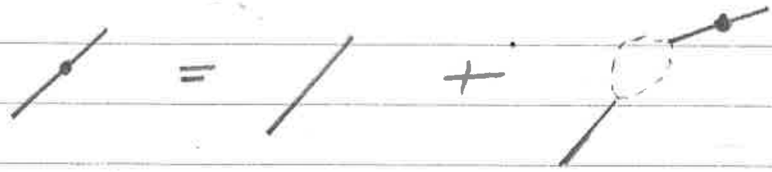
# Self-energy & Phase Space & Cutkosky Rule



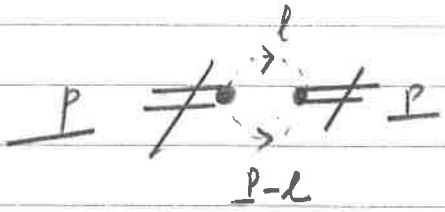
$$iG \qquad iG_0 \qquad iG_0 \, (-i\Sigma) \, iG$$

$$G^{-1} = G_0^{-1} - \Sigma$$

$$-i\Sigma_{\underline{p}} = \int \frac{d^4 l}{(2\pi)^4} \, iG_{l_1} \, iG_{l_2} \, (-ig)^2$$

$$l_1 = l$$

$$l_2 = \underline{p} + l$$




$$\Sigma(\underline{p}) = i g^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l_1^2 - m_1^2 + i\epsilon} \frac{1}{l_2^2 - m_2^2 + i\epsilon}$$

KEY STEP #1

$$\longrightarrow \frac{g^2}{16\pi^2} \int_0^1 dx \, \ln \frac{\Delta}{\Lambda^2} + C$$

$$\Delta = \left[x m_1^2 + (1-x) m_2^2 - x(1-x) \underline{p}^2 - i\epsilon\right]$$

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# KEY STEP #2

$$\mathrm{Im}\,\Sigma = -\frac{1}{2} g^2 \, \ell_2 \times \text{sym}$$

~~to relate to width~~

$$- \sqrt{s}\, \gamma(\sqrt{s})$$

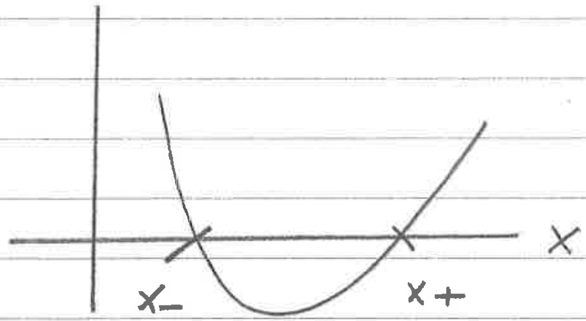
$$\rightarrow \frac{g^2}{16\pi^2} \int_0^1 dx \; -\pi \; \theta\left[ x(1-x)p^2 - (x m_1^2 + (1-x) m_2^2) \right]$$

$$= \frac{-1}{16\pi} g^2 \int_{x_-}^{x_+} dx$$

if $m_1 = m_2$ the condition is

$$x(1-x)s - m^2 > 0$$

$$x^2 - x + \frac{m^2}{s} < 0$$




$$x_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4m^2}{s}}\right)$$

2 if $m_1 = m_2$

$$x_+ - x_- = \sqrt{1 - \frac{4m^2}{s}}$$

$$\mathrm{Im}\,\Sigma = \frac{-1}{16\pi} g^2 \sqrt{1 - \frac{4m^2}{s}} \times \text{sym}$$

if $m_1 = m_2$

$$\rightarrow \frac{-1}{2} g^2 \, \ell_s \times \text{sym}$$

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<u>key result 1</u>

$$\frac{1}{D_1 D_2} = \int_0^1 dx \frac{1}{[x D_1 + (1-x) D_2]^2}$$

proof:

$$\frac{1}{D_1 D_2} = \left(\frac{1}{D_2} - \frac{1}{D_1}\right) \frac{1}{D_1 - D_2}$$

$$= \frac{1}{D_1 - D_2} \int_{D_1}^{D_2} d\tilde{x} \frac{-1}{\tilde{x}^2}$$

$$\tilde{x} = D_1 + x(D_2 - D_1)$$

$$\rightarrow \int_0^1 dx \frac{1}{[D_1 + x(D_2 - D_1)]^2}$$

$x' \to 1-x$

$$\text{or} \int_0^1 dx' \frac{1}{[x' D_1 + (1-x') D_2]^2}$$

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$$\Sigma_P = i g^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l_1^2 - m_1^2 + i\delta} \frac{1}{l_2^2 - m_2^2 + i\delta}$$

$$= i g^2 \int_0^1 dx \frac{d^4 l}{(2\pi)^4} \, x$$

$$\frac{1}{\left[x(l_1^2 - m_1^2) + (1-x)(l_2^2 - m_2^2)\right]^2}$$

$\swarrow$

$l_1 = l$

$l_2 = P - l$

$$x l^2 - x m_1^2$$

$$+$$

$$(1-x)(l^2 + P^2 - 2 l \cdot P) - (1-x) m_2^2$$

$\swarrow$

$$[l' - (1-x)P]^2 - \Delta$$

$$\Delta = x m_1^2 + (1-x) m_2^2 - x(1-x) P^2 - i\delta$$

$$\to i g^2 \int dx \int \frac{d^4 l''}{(2\pi)^4} \frac{1}{[l''^2 - \Delta]^2}$$

$d^4 l'' \to i d^4 l_E \qquad l''^2 \to - l_E^2 = (-l_1^2 - l_2^2 - l_3^2 - l_4^2)$

$l_0'' \to i l_4$

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$$\Sigma_p = -g^2 \int_0^1 dx \int \frac{d^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2+\Delta)^2}$$

Schwinger

$$\Rightarrow \frac{1}{A^2} = \int_{1/\Lambda^2}^{\infty} dt \; t \; e^{-tA}$$

$$\Sigma_p \longrightarrow -g^2 \int dx \int \frac{d^4 l_E}{(2\pi)^4} \int dt \; t \; e^{-t(l_E^2+\Delta)}$$

$$= -g^2 \int dx \int dt \; t \; e^{-t\Delta} \times \frac{1}{(2\pi)^4} \left(\sqrt{\frac{\pi}{t}}\right)^4$$

$$= \frac{-g^2}{16\pi^2} \int dx \underbrace{\int_{1/\Lambda^2}^{\infty} dt \; \frac{1}{t} e^{-t\Delta}}_{-\ln\frac{\Delta}{\Lambda^2} + C}$$

$$\Delta = x m_1^2 + (1-x) m_2^2 - x(1-x)p^2 - i\epsilon$$

useful results on
Schwinger proper time regularization :

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$$A^{-1} = \int_0^\infty dt\ e^{-tA}$$

$$\ln A = -\int_0^\infty dt\ \frac{1}{t}\left(e^{-tA} - e^{-tI}\right)$$

$$A^{-2} = \int_0^\infty dt\ t\ e^{-tA}$$

* going further :
$\mathrm{Im} \rightarrow \mathrm{Real}$
by dispersive technique

finally

$$\Sigma_P = \frac{g^2}{16\pi^2}\int dx\ \ln\frac{\Delta}{\Lambda^2} \times \underbrace{\mathrm{sym}}$$

2 if $m_1 = m_2$

$$\Delta = x m_1^2 + (1-x) m_2^2 - x(1-x) P^2 - i\delta$$

key result
\# 1

//

$$\frac{1}{A_1 A_2 \cdots A_N} = (N-1)! \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_N \frac{\delta(1 - x_1 - x_2 - \cdots - x_N)}{(x_1 A_1 + x_2 A_2 + \cdots + x_N A_N)^N}$$

see wiki

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key result 2

$$\mathrm{Im}\,\Sigma \leftrightarrow \qquad -\frac{1}{2} g^2 \,\mathcal{C}_S \quad \times \quad \text{sym}$$

$$\Sigma = i g^2 \int \frac{d^4 l_1}{(2\pi)^4} \frac{1}{l_1^2 - m_1^2 + i\delta} \frac{1}{l_2^2 - m_2^2 + i\delta}$$

$$\downarrow \; l_2 = p - l_1$$

$$= i g^2 \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \frac{1}{l_1^2 - m_1^2 + i\delta} \frac{1}{l_2^2 - m_2^2 + i\delta} \times (2\pi)^4 \delta^4 (p - l_1 - l_2)$$

$$= i g^2 \int_{l_1^0, l_2^0} \frac{d^3 l_1}{(2\pi)^3} \frac{d^3 l_2}{(2\pi)^3} dx^0 \frac{dl_1^0}{2\pi} \frac{dl_2^0}{2\pi} \frac{1}{l_1^{0\,2} - \varepsilon_1^2 + i\delta} \frac{1}{l_2^{0\,2} - \varepsilon_2^2 + i\delta} e^{i x^0 (p^0 - l_1^0 - l_2^0)} (2\pi)^3 \delta^3 (\vec{p} - \vec{l}_1 - \vec{l}_2)$$

$$\downarrow \; x^0 > 0 \text{ only}$$

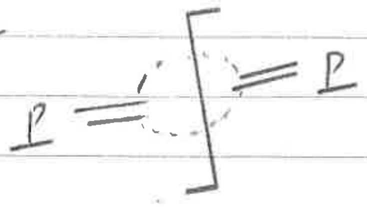
/ see tutorial notes /

$$= i g^2 \int \frac{d^3 l_1}{(2\pi)^3} \frac{d^3 l_2}{(2\pi)^3} \frac{-i}{2\varepsilon_1} \frac{-i}{2\varepsilon_2} (2\pi)^3 \delta^3 (\vec{p} - \vec{l}_1 - \vec{l}_2) \times$$

$$\int_0^\infty dx^0 \, e^{i x^0 (p^0 - \varepsilon_1 - \varepsilon_2)} \leftrightarrow \frac{1}{2} (2\pi \delta_{p^0 - \varepsilon_1 - \varepsilon_2}$$

$$\Rightarrow \mathrm{Im}\,\Sigma = -\frac{1}{2} g^2 \,\mathcal{C}_2 \quad //$$

## Cutkosky Rule

$-2\,\mathrm{Im}$  $= \int d\Omega_2 \left|\right.$  $\left.\right|^2 \times$ Sym fac.

$g^2$

motivates the study of

$$\mathcal{C}_2(s, m_1^2, m_2^2) = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{1}{4\varepsilon_1\varepsilon_2}\, 2\pi\,\delta(P^0-\varepsilon_1-\varepsilon_2) \times$$
$$(2\pi)^3\,\delta^3(\vec{P}-\vec{p}_1-\vec{p}_2)$$

the qty. depends only on $\underline{s = P^2 = P^{0\,2} - \vec{P}^2}$

via combination

CM frame:

$$P^0 = \sqrt{s} \qquad \vec{P} \to \vec{0}$$

$$\mathcal{C}_2(s) \to \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{4\varepsilon_1\varepsilon_2}\, 2\pi\,\delta\left(\sqrt{s} - \varepsilon_1 - \sqrt{p_1^2+m_2^2}\right)$$

$$\left.\frac{1}{\frac{p_1}{\varepsilon_1}+\frac{p_1}{\varepsilon_2}}\;\delta(p_1-q)\right|_{p_1\to q}$$

$$\mathcal{C} = \frac{1}{4\pi}\,\frac{q^2}{\varepsilon_1\varepsilon_2}\,\frac{1}{\frac{q}{\varepsilon_1}+\frac{q}{\varepsilon_2}}$$

$$= \frac{q}{4\pi\sqrt{s}} \quad // \qquad q = \frac{1}{2}\sqrt{s}\,\sqrt{1-\frac{(m_1-m_2)^2}{s}}\,\sqrt{1-\frac{(m_1+m_2)^2}{s}} \quad //$$

3-body phase space

$$\Phi_3(s, m_1^2, m_2^2, m_3^2) = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3} \frac{1}{8\,\varepsilon_1\varepsilon_2\varepsilon_3}$$

$$(2\pi)\,\delta_{\underline{p}^0 - \varepsilon_1 - \varepsilon_2 - \varepsilon_3}$$

$$(2\pi)^3\,\delta^3_{\underline{\vec{p}} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3}$$

Introduce :

$$1 = \int \frac{d\varepsilon'}{2\pi}\frac{d^3p'}{(2\pi)^3}(2\pi)^4\,\delta_{\varepsilon' - \varepsilon_1 - \varepsilon_2}\,\delta^3_{\vec{p}' - \vec{p}_1 - \vec{p}_2}$$

s.t.

$$\Phi_3 \rightarrow \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d\varepsilon'}{2\pi}\frac{d^3p'}{(2\pi)^3}$$

$$\frac{1}{4\varepsilon_1\varepsilon_2}\frac{1}{2\varepsilon_3}(2\pi)^4\,\delta_{\underline{p}^0 - \varepsilon' - \varepsilon_3}\,\delta^3_{\underline{\vec{p}} - \vec{p}' - \vec{p}_3}$$

$$(2\pi)^4\,\delta_{\varepsilon' - \varepsilon_1 - \varepsilon_2}\,\delta^3_{\vec{p}' - \vec{p}_1 - \vec{p}_2}$$

$$\longleftrightarrow \Phi_2(s'_{12}, m_1^2, m_2^2)$$

$$\longrightarrow \int \frac{d\varepsilon'}{2\pi}\frac{d^3p'}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\Phi_2(s'_{12}, m_1^2, m_2^2)\frac{1}{2\varepsilon_3}$$

$$(2\pi)^4\,\delta_{\underline{p}^0 - \varepsilon' - \varepsilon_3}\,\delta^3_{\underline{\vec{p}} - \vec{p}' - \vec{p}_3}$$

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Go to CM frame $\vec{P} = \vec{0}$ $P^0 \to \sqrt{s}$

$$\Phi_3(s) = \int \frac{d\varepsilon'}{2\pi} \frac{d\vec{p}'}{(2\pi)^3} \Phi_2(s_{12}', m_1^2, m_2^2)$$

$$\frac{1}{2\varepsilon_3} \quad 2\pi\,\delta(\sqrt{s} - \varepsilon' - \varepsilon_3)$$

$$d\varepsilon'\, 2\varepsilon' \frac{1}{2\varepsilon'} \qquad \sqrt{\vec{p}'^2 + \sqrt{s_{12}'}^2}$$

$$ds_{12}'$$

$$= \int_{(m_1+m_2)^2}^{(\sqrt{s}-m_3)^2} ds_{12}' \frac{1}{2\pi} \Phi_2(s, s', m_3^2)\, \Phi_2(s', m_1^2, m_2^2)$$

//

$$\Phi_N = \int \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_N}{(2\pi)^3} \left\{ \frac{1}{2\varepsilon_1} \cdots \frac{1}{2\varepsilon_N} \right\}$$

$$(2\pi)^4 \delta^4(P - \Sigma p_i)$$

introducing:

$$1 = \int \frac{d\varepsilon'}{2\pi} \frac{d^3p'}{(2\pi)^3} (2\pi)^4 \delta^4$$

$N-1$ subsystem

$$\varepsilon' = \sqrt{\vec{p}_N^2 + \sqrt{s}'^2}$$

$$d\varepsilon'^2 \to ds'$$

CM frame

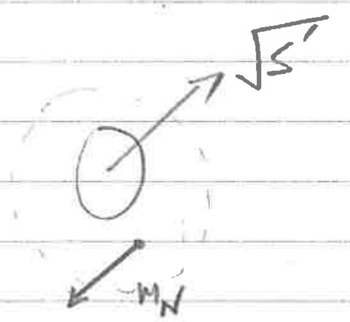
$$\ell_N = \int_{s'_-}^{s'_+} ds' \; \frac{1}{2\pi} \, \ell_2(s, s', m_N^2) \; \ell_{N-1}(s', m_1^2, \dots, m_{N-1}^2)$$

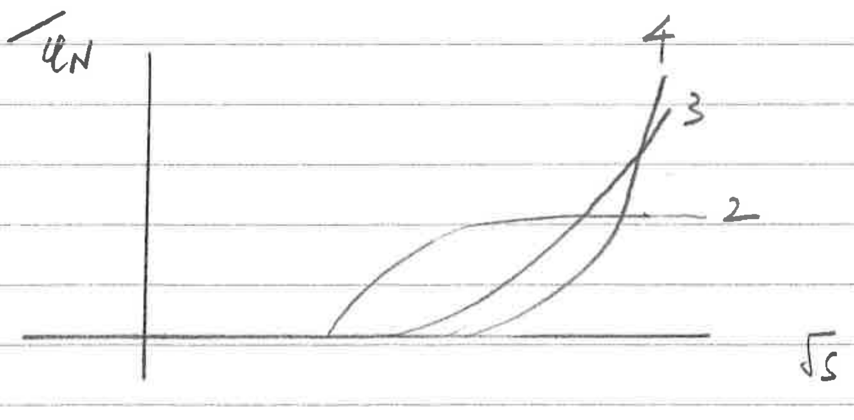
//

$$s'_- = (m_1 + m_2 + \dots + m_{N-1})^2$$

$$s'_+ = (\sqrt{s} - m_N)^2$$

good recursion formula to get the phase space




$$\ell_N \sim E^{2N-4}$$

unfortunate sign :

+ : for Peskin & Schroeder

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$$S = I + \underline{i\, T_E^{QFT}} \qquad (QFT)$$

Unitarity dictates :

$$\hookrightarrow \quad i T_E^{QFT} \leftrightarrow (2\pi)^4 \delta^4 \; \underline{i M^E}$$

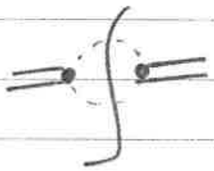
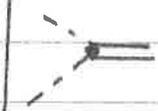
Feynman Rule & on-shell

$$S S^\dagger = I = I + i\left(T_E - T_E^\dagger\right) + |T_E|^2$$

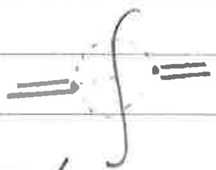
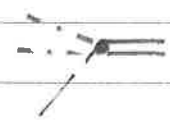
$$\Rightarrow \quad 2\,\mathrm{Im}\, T_E = |T_E|^2$$

$$i T_E \leftrightarrow -i \Sigma$$

$$-2\,\mathrm{Im}\, \Sigma = g^2 \Phi_2$$

$-2\,\mathrm{Im}$  $= \int d\Phi_2 \Big|$  $\Big|^2$ //

The amazing thing is that w/o calculation

$-2\,\mathrm{Im}$  $= \int d\Phi_3 \Big|$  $\Big|^2$

$$\Sigma = g_3^2 \int \frac{d^4\ell_1}{(2\pi)^4} \frac{d^4\ell_2}{(2\pi)^4} \, G_1 G_2 G_3 \left[(-i)^2 i^3 i\right]$$

2 loops result

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→ after a tedious calculation ...

2 Feynman parameters ...

$$-\mathrm{Im}\,\Sigma \rightarrow g_c^2 \frac{1}{(4\pi)^4} \int_0^1 dw \int_0^1 d\xi \, \frac{1-w}{\{(1-w)^2 \xi (1-\xi) + w(1-w)\}^2} \, \mathrm{Im}(\Delta \ln \Delta)$$

$$\Delta = M^2 - \frac{w(1-w)}{1-w+\frac{w}{\xi(1-\xi)}} p^2$$

& this monster equals

$$\Longrightarrow \frac{1}{2} g_3^2 \, \mathcal{E}_3(s)$$

note

$$-\mathrm{Im}\,\Sigma \leftrightarrow \sqrt{s}\,\gamma$$

Details in SMAT ntbk #1

tutorial notes : $\frac{1}{2}$ in <u>Cutkosky Rule</u>

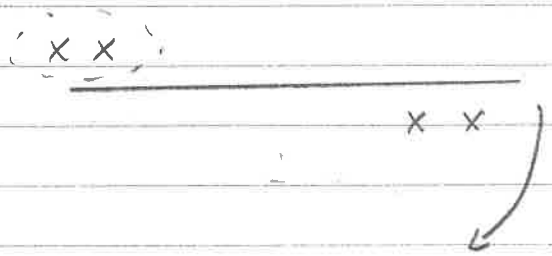
PML - pole method :

$$\int \frac{dl_1^0}{2\pi} \frac{dl_2^0}{2\pi} \frac{1}{l_1^{0\,2} - \mathcal{E}_1^2 + i\delta} \frac{1}{l_2^{0\,2} - \mathcal{E}_2^2 + i\delta} \; 2\pi\, \delta_{p^0 - l_1^0 - l_2^0}$$

$$\updownarrow$$

$$\frac{1}{2} \times \frac{-i}{2\mathcal{E}_1} \frac{-i}{2\mathcal{E}_2} \; 2\pi\, \delta_{p^0 - \mathcal{E}_1 - \mathcal{E}_2}$$

$l_1^0 \quad l_2^0$



$$2\pi\, \delta_{p^0 - l_1^0 - l_2^0} \longrightarrow \int_{-\infty}^{+\infty} dt \; e^{i(p^0 - l_1^0 - l_2^0)t}$$

even in $p^0 - l_1^0 - l_2^0$

close down okay <u>only</u> if $t > 0$

$l_1^0 \to -i\infty$.

$$\int_0^{\infty} dt \; e^{i(p^0 - \mathcal{E}_1 - \mathcal{E}_2)t}$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} dt \; e^{i(p^0 - \mathcal{E}_1 - \mathcal{E}_2)t}$$

$$= \frac{1}{2} \times 2\pi\, \delta_{p^0 - \mathcal{E}_1 - \mathcal{E}_2} \quad //$$

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to make it explicit

$$\int_{-\infty}^{+\infty} dt \, e^{it\Delta E} = \delta(\Delta E)$$

only real part contributes as RHS is real.

& real part is even in $t$

$$\Rightarrow 2\int_{0}^{\infty} dt \, \underline{\mathrm{Re}\, e^{it\Delta E}} = \delta(\Delta E)$$

$$\int_{0}^{\infty} dt \, e^{it\Delta E} = \frac{1}{2}\delta(\Delta E)$$

//

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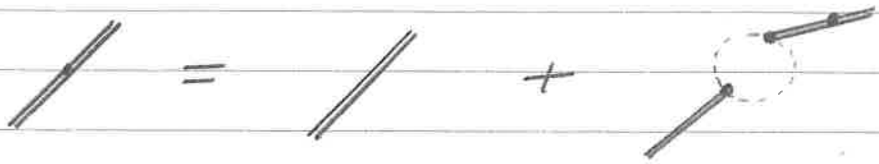
Self Energy, Decay & width



$$iG_{res} = iG^0_{res} + iG^0_{res}\,(-i\Sigma_{res})\,iG_{res}$$

$$G_{res} = \frac{1}{G^{0\,-1}_{res} - \Sigma_{res}}$$

$$G^{0\,-1}_{res} = p^2 - m^{0\,2}_{res} + i\delta$$

let's go to the rest frame of resonance:

$$G_{res} = \frac{1}{E^2 - m^{0\,2}_{res} - \Sigma_{res}(E, \vec{p}=\vec{0})}$$

$$\rightarrow \frac{1}{E^2 - \underline{\bar{m}^2_{res}} - i\,\mathrm{Im}\,\Sigma_{res}(E)}$$

finite after renorm.

$$\mathrm{Im}\,\Sigma_{res} \longleftrightarrow -E\gamma_E$$

$$= \frac{1}{E^2 - \bar{m}^2_{res} + iE\gamma_E}$$

use atan2

$$\theta_{res} \rightarrow \tan^{-1}\frac{E\gamma_E}{\bar{m}^2_{res} - E^2}$$

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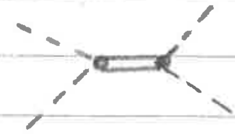
to discover this resonance from scattering experiment



$$iM_I = -ig \; i\Delta_{HS} \; -ig$$

$$iT_I \leftrightarrow iM_I \,\varphi_2$$

$$= i \frac{-g^2 \varphi_I}{E^2 - \bar{m}_{HS}^2 + E\gamma_I \, i}$$

The width $\rightarrow$

$$-\operatorname{Im}\Sigma = E\gamma_I = \frac{1}{2} g^2 \varphi_I$$

$$\gamma(E) = \frac{1}{2E}\int d\varphi_2 \, g^2$$

$$\rightarrow \frac{1}{2\sqrt{s}}\int d\varphi_2 \left| T_{HS\to p_1 p_2} \right|^2 \quad \text{— general}$$

$$G_{HS} \propto \frac{1}{E - \bar{m}_{HS} + \frac{1}{2}\gamma_E \, i}$$

t-space

$$G_{HS}(t) \longrightarrow e^{-i(\bar{m} - \frac{1}{2} i \gamma_E)t}$$

$$e^{-i\bar{m}t} \; e^{-\frac{1}{2}\gamma_E t}$$

$$P_{HS} \sim e^{-\gamma_E t}$$

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$$iT_E \leftrightarrow i \frac{-g^2 \gamma_E}{E^2 - \bar{m}_{res}^2 + E\gamma_E i}$$

$$= i \frac{-2E\gamma_E}{E^2 - \bar{m}_{res}^2 + E\gamma_E i} \approx i \frac{-\gamma_E}{E - \bar{m}_{res} + i\frac{1}{2}\gamma_E}$$

$$= 2i \sin\theta_{res} \; e^{i\theta_{res}} \quad // \qquad \text{checks !}$$

$$1 + iT_E = e^{2i\theta_{res}} = 1 + 2iq f_E$$

$$\Rightarrow \theta_{res} = \tan^{-1} \frac{E\gamma_E}{\bar{m}_{res}^2 - E^2} \quad //$$

note that

$$2\,\mathrm{Im}\, T_E = |T_E|^2$$

$$f_E^{res} = \frac{1}{2q} \left( \frac{-\gamma_E}{E - \bar{m}_{res} + i\frac{1}{2}\gamma_E} \right)$$

resonance parametrization

$$T_E \leftrightarrow 2q f$$

Kapusta

P. 251

$$\frac{d\sigma}{d\Omega} = |f|^2$$

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LS eq1.
$V + V G_0 T$

Scattering convention :

$$\hat{S} = I - i\, 2\pi \delta_{E-H_0} \hat{T}$$

$$\mathrm{tr} \ln \hat{S} \longleftrightarrow 2i\theta \longleftrightarrow \ln(1 - i\, \mathrm{tr}\, \hat{T}\, \ell)$$

$$e^{2i\theta} \leftrightarrow 1 - i \ell T \leftrightarrow 1 + 2iqf$$

$T$ here is really the T-matrix

$$\ell \to \frac{q}{4\pi\sqrt{s}}$$

$$T = \frac{-2qf}{\ell}$$

$$= -8\pi\sqrt{s} f$$

Standard result

compare ...

$$f_{NR} \to \frac{-1}{4\pi} 2m_R T$$

$V < 0$ attractive
$f > 0$
$\Sigma < 0$

$$\Rightarrow \Sigma \sim -4\pi f\, n \sim 2m_R T\, n$$

$$T_{scat} \sim -\mathcal{M}_E^{QST}$$

$S = I - i\phi T_{scat}$

Feynman Rules

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Summary :

$$-i\Sigma = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l_1^2 - m_1^2 + i\delta} \frac{1}{l_2^2 - m_2^2 + i\delta} g^2$$

$$\Rightarrow \quad \Sigma(p) = \frac{g^2}{16\pi^2} \int_0^1 dx \ \ln \frac{\Delta}{\Lambda^2} + c$$

$$\Delta = x m_1^2 + (1-x) m_2^2 - x(1-x) p^2 - i\delta$$

$$\mathrm{Im}\,\Sigma(p) = -\frac{1}{16\pi} g^2 \sqrt{1 - \frac{(m_1+m_2)^2}{s}} \sqrt{1 - \frac{(m_1-m_2)^2}{s}}$$

$$= -\frac{1}{2} g^2 \varphi$$

$$\varphi = \frac{q}{4\pi\sqrt{s}} \quad \text{where}$$

$$q = \frac{1}{2}\sqrt{s} \sqrt{1 - \frac{(m_1+m_2)^2}{s}} \sqrt{1 - \frac{(m_1-m_2)^2}{s}}$$

also directly from Cutkosky Rule

$$-\mathrm{Im}\,\Sigma(p) = \frac{1}{2}\int d\varphi_2 \, g^2 = \sqrt{s}\ \gamma(s)$$

$$\gamma(s) = -\frac{1}{\sqrt{s}} \mathrm{Im}\,\Sigma$$

$$= \frac{1}{2\sqrt{s}} \int d\varphi_2 \, g^2 \quad //$$

Resonance

No.

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$$S = 1 - i\varepsilon T_{scat} = 1 + i\phi M_E$$

$$iM_E = -ig^2 \frac{1}{E^2 - \bar{m}_{res}^2 - i\, Im\Sigma(E)}$$

Feynman rule

$$iM_E \phi = 2i g f = S - 1 = 2i \sin\delta\, e^{i\delta}$$

$$-Im\Sigma = E\gamma = \frac{1}{2} g^2 e$$

$$\tan\delta = \frac{-E\gamma}{E^2 - \bar{m}_{res}^2}$$

$$\simeq \frac{-\gamma/2}{E - \bar{m}_{res}}$$

//

$$f_{Rec} = \frac{1}{2g} \left( \frac{-2E\gamma}{E^2 - \bar{m}_{res}^2 + iE\gamma} \right)$$

$$\simeq \frac{1}{2g} \frac{-\gamma}{E - \bar{m}_{res} + i\frac{1}{2}\gamma}$$

$$\frac{d}{d\Omega}\sigma \sim |f|^2$$